A Figure-eight Hysteresis Pattern in Macroscopic Fundamental Diagrams for an Urban Freeway Network in Beijing, China

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ABSTRACT

This paper presents Macroscopic Fundamental Diagrams (MFDs) for an urban freeway network in Beijing, China. In the diagrams, a figure-eight hysteresis pattern is observed. To understand the causes, analyses are made ranging from spatial-temporal heterogeneity of vehicles to the flow-occupancy relation for individual locations. Eventually, at individual locations we observe that free-flow traffic with the same occupancy exhibits different flows in the onset and offset of a rush hour; it is attributed to the counter-clockwise loop in the figure-eight hysteresis pattern at the macroscopic level. Different lane-changing rates in the onset and offset of a rush hour are discussed as the deeper causes of the multi-branch flow-occupancy diagram at individual locations; it is closely related to the denseness of ramps on the urban freeway network in Beijing. The paper enriches the knowledge about MFDs and provides some empirical support for the existing theory.
INTRODUCTION

Nowadays, most of approaches of traffic management and control still highly rely on traffic data that are difficult to be obtained sometimes. Combined with complexity of traveler behavior and network topology, the practical effects are compromised. A recently proposed macroscopic fundamental diagram (MFD) for a large urban area provides a new thought on aggregate traffic management and control that are less affected by details.

Understanding the shape and characteristics of an MFD for a network is basic and significant to take advantage of the diagram in practice. In the MFDs for an urban freeway network in Beijing, China, we observe a figure-eight hysteresis pattern combining clockwise and counter-clockwise loops, which is only theoretically mentioned in Gayah and Daganzo (1). Accompanying with the counter-clockwise loop, lower occupancy variance is associated with lower mean flow; it is inconsistent with the observation in Geroliminis and Sun (2). The paper is dedicated to reporting the MFDs with the figure-eight hysteresis pattern, and to investigating the causes of the counter-clockwise loop and the association between lower occupancy and lower mean flow.

The remainder of the paper is organized as follows: literature review, the urban freeway network in Beijing and the data used are presented in the next two sections; these are followed by a presentation of the figure-eight hysteresis pattern and an investigation of the formation of the counter-clockwise loop; discussion and conclusions are made at last.

LITERATURE REVIEW

Investigations regarding relationships between macroscopic variables in an urban area could be traced back to Godfrey (3). In the literature the relationship between average speed and vehicle density was explored in a macroscopic view. The two-fluid model based on the fraction of moving and standing traffic was later addressed in Herman and Prigogine (4) and Herman and Ardekani (5). Some literature, e.g. Ardekani and Herman, Mahmassani et al., Mahmassani and Peeta, Olszewski and Fan (6, 7, 8, 9), also investigated the aggregate traffic relationships.

More recently, Daganzo (10) proposed an MFD that reflected invariant macroscopic relationships among space-mean flow, density and speed in a large urban area. Daganzo and Geroliminis (11) theoretically proved the existence of the MFD using variational formulation of the kinematic wave theory (see Daganzo, Daganzo (12, 13)) , and conjectured four regularity conditions ensuring a well-defined (low scatter) MFD. Meanwhile, Helbing (14) also derived analytical solutions for the MFD by using a utilization-based approach (see Daganzo (15)). Empirical evidence was provided by Geroliminis and Daganzo (16), in which data collected from an urban area in Yokohama, Japan, was used and a well-defined MFD was first observed.

A number of investigations regarding the MFD were conducted theoretically and practically since the seminal papers were released. In empirical study, Buisson and Ladier (17) first reported hysteresis phenomena with clockwise loops existing in an MFD for the Toulouse road network in France, and showed that heterogeneity in types and topology of road networks as well as locations of detectors had strong impacts on the shape of the MFD. Geroliminis and Sun (2) explicitly investigated causes of the clockwise hysteresis loops by utilizing data collected from the Twin Cities metropolitan area freeway network in Minnesota, USA. Two reasons of the clockwise loops were unveiled: different spatial-temporal distributions of congestion in the onset and offset, and synchronized occurrence of capacity drop at individual locations. An association between higher occupancy variance and lower mean flow was also observed. Geroliminis and Sun (18) compared the MFDs for the urban areas in Yokohama and Twin Cities, and analyzed characteristics of the
road network presenting a well-defined MFD. A sufficient existence condition for a well-defined MFD was addressed. It was also indicated that surface networks more likely exhibited MFDs with low scatter due to the characteristics of network redundancy, traffic signals, etc. To the authors’ knowledge, only the MFDs for the three cities have been reported. More empirical observations either supporting or contradicting existing findings are still expected to enrich the understanding of the MFD.

In analytical study, Daganzo (19) modeled traffic dynamics on a ring freeway with on- and off-ramps by using the kinematic wave theory. The model illustrated how the distribution of flow and density became uneven in the offset of a rush hour even when the ring was symmetric and the demand was uniform. Clockwise hysteresis loops arose with the unevenness. Daganzo and Gayah (20) modeled a square grid by using a two-ring idealization and further simplified into a two-bin model. The results showed that random turning at intersections aggravated congestion and thereby led to uneven congestion and hysteresis phenomena in the MFD. Gayah and Daganzo (1) incorporated trip ends into the two-bin model and came to a conclusion that traffic usually exhibited more instability in the offset of a rush hour than in the onset; it also implied that hysteresis phenomena could also arise due to occurrence of unexpected disturbance even in a symmetric network with uniform demand. Meanwhile, the literature illustrated a figure-eight hysteresis pattern, and stated that the pattern occurred if the loading demand was very unbalanced and the maximum density was quite high; the conditions were rare and no empirical observation has been reported yet.

To provide more state-of-the-art information, we keep reviewing the simulation and application study, although this paper doesn’t belong to the types of study. In simulation study, Mazloumian et al. (21) proposed a traffic flow simulation model based on the section-based traffic model (see Helbing (22)). A variety of simulation scenarios were conducted, and the spatial distribution of vehicles measured by variability of vehicle densities was considered as a key variable of traffic performance and the scatter in the MFD. Knoop and Hoogendoorn (23) developed a road network simulation model based on the cell transition model, and further investigated the influence of the variability. A two-variable macroscopic fundamental diagram incorporating a dimension of the variability was suggested. Ji et al. (24) modeled the A10 west in Amsterdam, the Netherlands on VISSIM. The influence of various factors on the MFD was demonstrated, such as ramp-metering, the onset and offset of congestion, rapidly changing demands, etc.

In application study, Daganzo (10) proposed an accumulation-based (AB) rule for optimizing arrival rates of vehicles based on a given MFD. Gonzales et al. (25) demonstrated the applications of the AB rule via a simulation model of an urban area in San Francisco, USA. Perimeter control approaches could be used to implement the AB rule, such as modifying signal control, rationing license plate, etc. Interaction of multiple modes in the MFD scheme was also analyzed. Daganzo et al. (26) discussed similar issues in an example to show the benefits of parsimonious models. Zheng et al. (27) developed an MFD-controlled cordon pricing scheme. In the scheme, a toll was determined based on the MFD of the target network, and the objective was to maintain mean flow of the network at the maximum value of the MFD. Knoop et al. (28) attempted to apply the MFD in routing. A few of routing strategies were compared in a network simulation model. The results showed improved traffic flow, importance of properly partitioning network, etc. Haddad and Geroliminis (29), Remezani et al. (30) and Haddad et al. (31) partitioned an urban network into two regions each with an independent MFD: a city center and its periphery. Stability of the two-region system was analyzed and a few of optimal traffic control problems were explored.
URBAN FREEWAY NETWORK IN BEIJING AND THE DATA

Urban ring freeways in Beijing

Beijing is one of the largest cities in the world. At present, the urban area of Beijing is enclosed by four two-way urban ring freeways, i.e., the 2nd-5th rings. Among these, the 3rd ring with three lanes in each direction is 48.3 km and the speed limits are 80 km/h for straight sections and 60 km/h for curves. 74 Remote Transportation Microwave Sensors (RTMS) covering two-way traffic have been installed on the ring (see Figure 1). Traffic flow data (i.e., occupancy, flow and speed) used in the paper are collected on the ring from 6 am to 12 pm on four weekdays, i.e., June 3-6 (Mon-Thu), 2002. The data are aggregated every two minutes.

FIGURE 1 The urban freeway network in Beijing and locations of RTMS on the 3rd urban ring freeway.

Data processing

The data are processed as the following three steps and Table 1 presents the results:

1. **step 1**: eliminating ineffective RTMS by checking if a data file contains data;
2. **step 2**: observing the flow-occupancy diagram drawn by using data from each individual lane, and discarding entire data pertaining to a lane whose diagram looks obviously incorrect. In the step, the flow-occupancy relation and similarity of the diagrams for adjacent lanes are mainly considered. The step is manual and relies on basic traffic flow knowledge;
3. **step 3**: removing missing and out-range data. If one item in a data unit of occupancy, flow and speed at a time slice is out of given intervals, the unit will be removed. The intervals of occupancy, flow and speed are chosen as [0,100]%, [0,2500] veh/h and [0,100] km/h, respectively.
<table>
<thead>
<tr>
<th>Date</th>
<th>step 1: RTMS</th>
<th>step 2: lane data</th>
<th>step 3: data unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>total #</td>
<td>ineff. #</td>
<td>total #</td>
</tr>
<tr>
<td>June 3</td>
<td>74</td>
<td>9</td>
<td>419</td>
</tr>
<tr>
<td>June 4</td>
<td>74</td>
<td>9</td>
<td>419</td>
</tr>
<tr>
<td>June 5</td>
<td>74</td>
<td>11</td>
<td>407</td>
</tr>
<tr>
<td>June 6</td>
<td>74</td>
<td>9</td>
<td>419</td>
</tr>
</tbody>
</table>

**Building Macroscopic Fundamental Diagrams**

Since entire data from some lanes are excluded, we take an average of data of the rest of lanes in a direction covered by a RTMS (we regard a direction of each RTMS as a location in the rest of the paper) to represent traffic conditions at the location. Then, the space-mean flow and occupancy on the ring is derived by using the formula introduced in Geroliminis and Daganzo (16).

Specifically, let \( i \) and \( N \) be the index and the total number of locations covered by all effective RTMS, and denote by \( N_i \) the number of effective lanes at location \( i \). Occupancy is directly used without being converted to density as usual. Mean flow and mean occupancy at time interval \( k \) are obtained as follows:

\[
Q(k) = \frac{1}{N} \sum_{i=1}^{N} \bar{q}_i(k), \quad \bar{q}_i(k) = \frac{1}{N_i} \sum_{j=1}^{N_i} \alpha_{ij}(k)q_{ij}(k)
\]

\[
O(k) = \frac{1}{N} \sum_{i=1}^{N} \bar{o}_i(k), \quad \bar{o}_i(k) = \frac{1}{N_i} \sum_{j=1}^{N_i} \beta_{ij}(k)o_{ij}(k)
\]

where \( q_{ij}(k) \) and \( o_{ij}(k) \) are flow and occupancy collected on lane \( j \) at location \( i \) every two minutes; \( \alpha_{ij}(k), \beta_{ij}(k) \in \{0, 1\} \) are dummy coefficients that are equal to 1 if the data unit is effective; 0, otherwise.

**MACROSCOPIC FUNDAMENTAL DIAGRAMS FOR THE URBAN FREEWAY NETWORK IN BEIJING**

**Existence of a figure-eight hysteresis pattern**

MFDs for the four weekdays are built in Figure 2. Meanwhile, the global variance of occupancy among all locations in a time interval (denoted by \( V(k) \)) is calculated to represent spatial heterogeneity, and a relation between occupancy variance and mean occupancy is also plotted in the same figure.

In the figure, two distinguishing features could be observed: (i) figure-eight hysteresis combining clockwise and counter-clockwise loops; (ii) an association between lower occupancy variance and lower mean flow accompanying with the counter-clockwise loops; it is inconsistent with the observation in Geroliminis and Sun (2). The causes of the clockwise loops have been explicitly investigated theoretically and empirically in Daganzo (19), Buisson and Ladier (17) and Geroliminis and Sun (2); refer to the review of the papers. Therefore, the remainder of the paper concentrates on the formation of the counter-clockwise loop and the accompanied association.
FIGURE 2 Mean flow vs. mean occupancy (upper plot) and occupancy variance vs. mean occupancy (lower plot) for the 3rd ring on June 3-6, 2002 (the gradually changing colors from red to blue demonstrate the time growth from 6:00 am to 12:00 pm)

Formation of the counter-clockwise loop in the figure-eight hysteresis pattern

We select the counter-clockwise direction of the 3rd ring as an example and deeply look at its spatial-temporal heterogeneity of vehicles (note that there are two directions on a ring road, which are usually called the clockwise and counter-clockwise directions). Figure 3 first shows the relations between mean flow and mean occupancy and between occupancy variance and mean occupancy; the aforementioned features are also observed. Figure 4 combines time, locations and occupancy, and provides a clear look at the spatial-temporal heterogeneity. Heavy congestions occur at the locations around 3040 and 3070. The congestion around location 3070 vanishes at the end of the rush hour, while the congestion around location 3040 lasts to the end; it implies higher occupancy variance in the offset of the rush hour, and provides an empirical evidence that unevenness of vehicle distribution will arise in the offset of a rush hour on a ring road, which was theoretically stated in Daganzo (19).
FIGURE 3 Mean flow vs. mean occupancy (upper plot) and occupancy variance vs. mean occupancy (lower plot) for the counter-clockwise direction in the 3rd ring on June 4, 2002 (the gradually changing colors from red to blue demonstrate the time growth from 6:00 am to 12:00 pm)

FIGURE 4 Spatial-temporal heterogeneity of vehicles in the counter-clockwise direction of the 3rd ring from 6:00 am to 12:00 pm on June 4, 2002

To see more details, we select a pair of two-minute intervals on the counter-clock loop, i.e., \(k_1\) and \(k_2\), which start from 7:03 and from 11:45, respectively. In the paired time intervals the mean occupancy is approximate, while lower occupancy variance is associated with lower mean
flow, i.e., \( V(k_1) < V(k_2) \) and \( Q(k_1) < Q(k_2) \), when \( O(k_1) \approx O(k_2) \). We plot the occupancy, flow, and speed in each time interval at all locations in Figure 5. From the figure, two distinguishing traffic conditions can be observed, those are, (a) the congested condition existing at location 3042 and 3043 (denoted by \( M \) the set of the two locations), where higher occupancy is associated with lower speed comparing with other locations; (b) the free-flow condition at the other locations (denoted by \( \overline{M} \)), where all occupancy, speed and flow are close. We now check the association:

\[
V(k_1) < V(k_2) \quad \text{and} \quad Q(k_1) < Q(k_2), \quad \text{when} \quad O(k_1) \approx O(k_2). \quad \text{(3)}
\]

**FIGURE 5** Traffic conditions in the two-minute intervals respectively starting from 7:03 and 11:45 in the counter-clockwise direction of the 3rd ring on June 4, 2002 (the blue: occupancy, the green: flow, and the red: speed)
To achieve $Q(k_1) < Q(k_2)$ and simultaneously $O(k_1) \approx O(k_2)$, other locations $\overline{M}$ should have

$$\sum_{i \in \overline{M}} q_i(k_1) < \sum_{i \in \overline{M}} q_i(k_2), \quad \sum_{i \in \overline{M}} o_i(k_1) > \sum_{i \in \overline{M}} o_i(k_2)$$

(4)

However, we can not see the relations in the figure, and they are rare to the free-flow condition, in which flow and occupancy at an individual location is usually positively correlated as well as the sum of flow and occupancy from different locations based on the fundamental traffic flow theory.

To understand the relations, flow-occupancy relations at different locations in the paired time intervals $k_1$ and $k_2$ are plotted in Figure 6. It can be seen in general that the flow in $k_1$ is smaller than that in $k_2$, and the occupancy in $k_1$ is greater than that in $k_2$, which lead to inequality (4). It provides insight into the cause of $Q(k_1) < Q(k_2)$ and simultaneously $O(k_1) \approx O(k_2)$.

The observation, however, is interesting: in the free-flow condition, the same occupancy in $k_1$ is associated with lower flow than that in $k_2$. To show no coincidence, we further present the flow-occupancy relations on other days; see Figure 7.

![Figure 6](image_url)

**FIGURE 6** Flow vs. occupancy at all locations at 7:03 and 11:45 on June 4, 2002 (The curves are fitted using data units which occupancy is lower than 20%)
FIGURE 7 Flow vs. occupancy at all locations at pairs of time slices with the approximate mean occupancy on June 3, 5 and 6, 2002 (The curves are fitted using data units which occupancy is lower than 20%)
(a) The location of RTMS 3002 in front of National Agriculture Exhibition Center of China

(b) Median lane, clockwise, 3002

(c) Center lane, clockwise, 3002

(d) Shoulder lane, clockwise, 3002

FIGURE 8 Flow-occupancy diagrams for location 3002 on the 3rd ring on June 4, 2002 (the gradually changing colors from red to blue demonstrate the time growth from 6:00 am to 12:00 pm)

(a) The location of RTMS 3072 on Sanyuan West Bridge

(b) Median lane, counter-clockwise, 3072

(c) Center lane, counter-clockwise, 3072

(d) Shoulder lane, counter-clockwise, 3072

FIGURE 9 Flow-occupancy diagrams for location 3072 on the 3rd ring on June 4, 2002 (the gradually changing colors from red to blue demonstrate the time growth from 6:00 am to 12:00 pm)
DISCUSSION OF THE MULTI-BRANCH FLOW-OCCUPANCY DIAGRAM

The urban freeway network in Beijing has unique characteristics, such as a large number of auxiliary roads surrounding and connecting with the urban freeways, dense ramps (a ramp per 0.5 km on average, approximately; refer to Figure 8(a) and 9(a) for examples), short ramp length, many interchanges, etc. All of these have significant impacts on the traffic and driver behavior, in particular the dense ramps, which result in frequent lane-changing maneuvers.

In recent work on the traffic on the urban freeways, we proposed an inhomogeneous macroscopic traffic flow model with a multi-branch fundamental diagram (see He et al., He and Guan (32, 33)). Such two free-flow branches could be explained as a result of different lane-changing rates in the onset and offset of a rush hour.

We briefly introduce the macroscopic driver perception (MDP) model proposed in He and Guan (33). The model extends the LWR model (34, 35) by considering a speed-density relation $u = U(\rho, \mu)$, where $u$ and $\rho$ are speed and density, respectively, and the diver perception factor $\mu$ changes with surrounding traffic situations. The equation of the MDP model reads:

$$\frac{\partial (u \mu)}{\partial t} + \frac{\partial (\rho u \mu)}{\partial x} = \psi \mu_e(\psi) + \frac{\rho u (\mu_e(\psi) - \mu)}{\epsilon},$$

where $\psi$ representing lane-changing frequency is a known function on space time point $(x, t)$, $\epsilon$ is a relaxation factor in units of distance, and $\mu_e(\psi)$ is a desired perception factor dependent on $\psi$. The flow $\rho u \mu$ is named as “perception flow” as $\mu$ is a driver perception factor. The first part $\psi \mu_e(\psi)$ in the source term of Equation 5 indicates the increased perception flow per distance unit caused by lane-changing vehicles with perception factor $\mu_e(\psi)$. The second part $\rho u (\mu_e(\psi) - \mu)/\epsilon$ means the increased perception flow per distance unit caused by vehicles in the target lane with perception factor $\mu$ and relaxes to $\mu_e(\psi)$ as lane-changing vehicles enter.

The perception-dependent speed-density relation $U(\rho, \mu_e(\psi))$ is constructed in the paper, and the multi-branch fundamental diagram is calibrated using the empirical data collected on a road section with dense ramps on an urban ring freeway in Beijing; see Figure 10. The solution to the corresponding Riemann problem is further provided, and numerical simulations show that the model is able to reproduce the patterns observed at on-ramp inhomogeneity.

Based on the multi-branch flow model, the lower free-flow branch in the onset of congestion is caused by higher lane-changing rates. In an urban freeway network with dense ramps, like the urban freeways in Beijing, locating detectors closely to ramps is difficult to be avoided, and more lane-changing maneuvers are thus included in the detected traffic flow data. Indeed, lane-changing maneuvers on the urban freeways are more frequent due to the denseness of ramps, and the kind of data describes the reality. It is obvious that the lane-changing maneuvers are closely related to inflow and outflow via on- and off-ramps as well as the OD matrices. Due to lack of the lane-changing data or the data on ramps, we can not directly show the change of the lane-changing rates at the study locations of the paper. However, it is not difficult to imagine that higher demands for the urban freeway in the onset of the rush hour result in higher inflow and higher lane-changing rates in the vicinity of on-ramps, and consequently lead to a lower free-flow branch in the flow-occupancy relations.
FIGURE 10 Empirical multi-branch fundamental diagrams: (a) flow-density curves, (b) speed-density curves.

CONCLUSIONS

A figure-eight hysteresis pattern is observed in the MFDs for the 3rd urban ring freeway in Beijing, China. To understand the causes, analyses are made ranging from spatial-temporal heterogeneity of vehicles to the flow-occupancy relation for individual locations. At individual locations, it is observed that the free-flow traffic with the same occupancy exhibits lower flow in the onset of the rush hour and higher flow in the offset. The multi-branch flow-occupancy relation at the microscopic level, consequently, results in the counter-clockwise loop in the figure-eight hysteresis pattern and the association between lower occupancy variance and lower mean occupancy at the macroscopic level.

It is discussed that the multi-branch flow-occupancy relation is caused by different lane-changing rates in the onset and offset of a rush hour. Frequent lane-changing maneuvers due to dense ramps are an important characteristic of the traffic on the urban freeway network in Beijing. Although more work is still needed to shed light on the phenomena, the results still indicate that both lane-changing rates (or detector locations) and the shape of the fundamental diagram for individual locations have significant impacts on the shape of the MFD.

Moreover, this paper presents the MFDs for an urban ring freeway. The hysteresis phenomena are also observed in the MFDs for the urban freeway network with more ramps (i.e., more route choices for drivers than regular freeways). Meanwhile, the results also provide empirical support that unevenness of the vehicle distribution will arise in the offset of a rush hour on a ring road.

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